Simulating elections under alternative electoral systems

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Introduction

Electoral system reform asks: **How would elections be different under a different electoral system**?

I estimate the results of a historical Single-Member District (SMD) election if it had occurred under Proportional Representation (PR) or Ranked Choice Voting (RV) instead.

The challenge: electors' choices can change in response to the electoral system. Using game theory models of vote choice, election results, and public opinion data, I simulate how voters from an SMD election might behave under PR.

I apply this method to the Canadian 2008 election, a close election that spawned a movement to reform Canada's SMD system (Figure 1). In 2017, a LIBERAL government rejected reform, publicly stating that they preferred RV to SMD, and SMD to PR. However, inferring rankings from public opinion data suggests that only some types of RV would benefit them, and modeling voter strategy shows that PR may benefit them.

Figure 1: Two news articles from the Canadian Broadcasting Corporation show the salience of reform.

If voters vote the same way under PR as SMD, then PR would harm the dominant or regionally dominant parties – the LIBERALS, CONSERVATIVES, and BLOC QUÉBÉCOIS (BLOC) – and help the NEW DEMOCRATIC PARTY (NDP) and GREENS. Figure 2 compares parties' vote proportions to their seats won in every election since WWII.
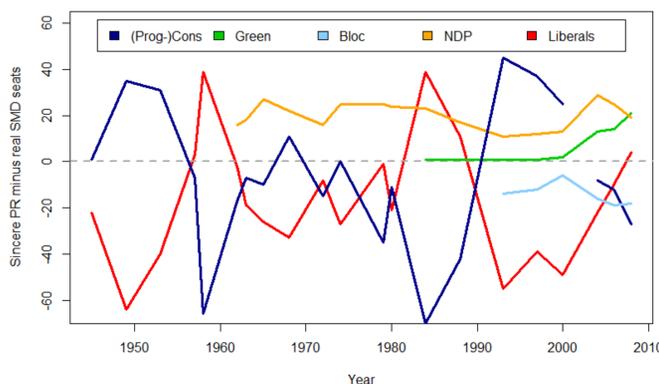

Figure 2: Gains or losses by party from switching to PR, if votes are not a function of the electoral system.

Sincere PR takeaway: Figure 2 validates the LIBERAL opposition to PR, if votes do not change with the electoral system.

To estimate a sincere RV election, I use public opinion data to impute preferences from vote choice. Using party rankings from the Canadian National Election Study (2008), I assume that:

$$P_s(r_j = \rho_k | r_i = \rho_1) = P_t(r_j = \rho_k | r_i = \rho_1)$$

where P_s is a proportion of survey respondents, P_t is a proportion of electors, r is any party, ρ is the party in a given position in a preference ordering, and with $j, k \neq 1$ and $i \neq j$. So if, for example, a respondent who likes the CONSERVATIVES most has probability 0.2 of liking the LIBERALS second-most, then a CONSERVATIVE voter also has probability 0.2

of liking the LIBERALS second-most.

With this assumption I calculate a Borda election where everyone has either 5, 4, 3, or 2 votes, and electors vote sincerely:
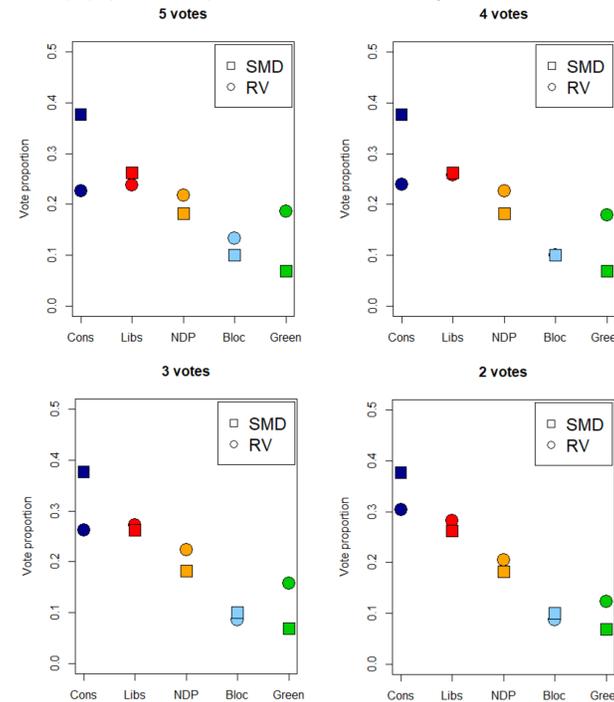

Figure 3: Gains or losses by party from switching to RV, if votes are not a function of the electoral system.

Sincere RV takeaway: LIBERALS should prefer RV with 2 or 3 votes – but not 4 or 5. The CONSERVATIVES and BLOC would lose seats from sincere RV, and the NDP and GREENS would gain seats.

Do these estimates hold with strategic voters?

Simulating strategic elections

Election simulation procedure

1. **Pick a voting rule**: I assume that Canadians in 2008 were maximizing their expected utility [1; 2], and that wasted votes come from people who hold both top 2 parties in last place.

2. **Generate an electorate**: The observed result must be a possible result of the electorate, with nobody voting for their last-ranked party. For example, in a district outside Québec where the top 2 parties were the LIBERALS and CONSERVATIVES:

   1: Set the number C who voted CON
   2: Set the number L who voted LIB
   3: Set the number N who voted NDP
   4: Set the number G who voted GREEN
   5: **for** 1 to C **do**
   6:     Generate an elector where CON is not last
   7: **end for**
   8: **for** 1 to L **do**
   9:     Generate an elector where LIB is not last
   10: **end for**
   11: **for** 1 to N **do**
   12:     Generate an elector with NDP not last, LIB/CON tied for last
   13: **end for**
   14: **for** 1 to G **do**
   15:     Generate an elector with GREEN not last, LIB/CON tied for last
   16: **end for**

3. **Validate the model**: Simulate the range of results that the electorate could have produced under SMD, and check it against the real results.

4. **Choose an electoral system**: I will simulate PR where electors vote in m-winner provinces, where m is the number of SMD districts there.

Validating the strategic model

If we simulate an SMD election, does the model reproduce the real results?

My simple preference estimation rule uses *national* data to impute *local* preference orderings, so the crucial test is how it performs in districts that don't resemble the national environment. I test it in a **similar district** – where the two top-performing parties are the LIBERAL and CONSERVATIVE parties – and in a **different district** – where the top-performers are the LIBERAL and NDP.

In the **similar district**, the model believes the CONSERVATIVES enjoy sizable support, with electors compromising and voting LIBERAL:
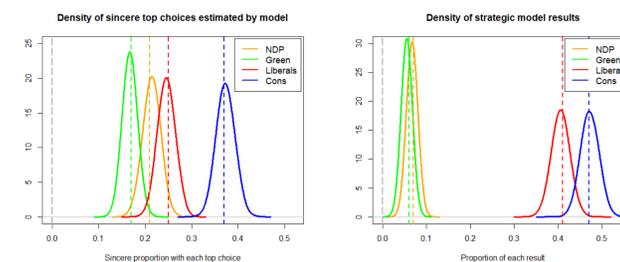

Figure 4: Left: Density of preferences estimated by the strategic model in Mississauga-Erindale, a district that is similar to the national 2008 election. Right: Strategic model results in Mississauga-Erindale.

The **similar district** simulation is nearly identical to the real results:
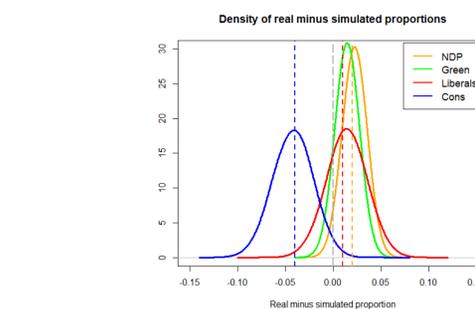

Figure 5: How close the strategic model results are to the real results in the similar district.

In the **different district**, despite many CONSERVATIVE supporters, only the previous election's NDP and LIBERAL winners are competitive:
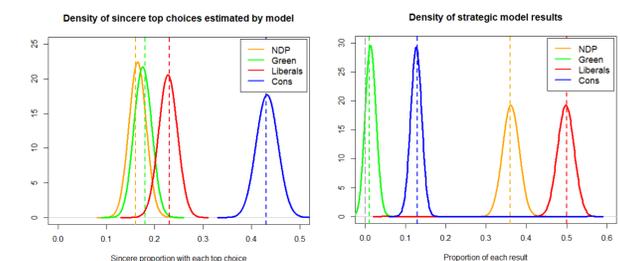

Figure 6: Left: Density of preferences estimated by the strategic model in Trinity-Spadina, a district that is different from the national 2008 election. Right: Strategic model results in Trinity-Spadina.

The preference imputation method gets the numbers in the **different district** wrong, but it captures the qualitative race:
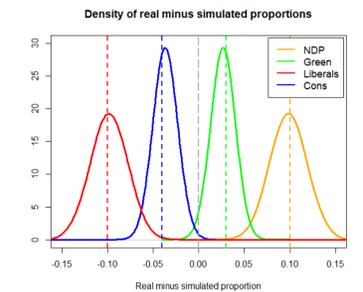

Figure 7: How close the strategic model results are to the real results in the different district.

Model validation takeaway: Even in an extremely challenging district, the electorate behaves like the real electorate does under SMD.

Strategic PR results

Now apply the strategic model to a PR election with m-winner provinces. The real CONSERVATIVES won a minority, beating the LIBERALS.
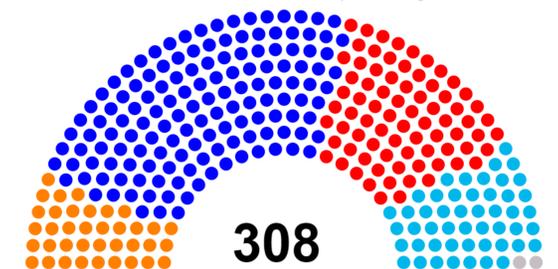

Figure 8: Real 2008: CONSERVATIVES 143 seats; LIBERALS 77; BLOC 49; NDP 37; GREENS 0; Ind 2.

Averaging 50 simulations, the LIBERALS manage a minority, picking up some BLOC and CONSERVATIVE voters. The NDP gain 14 seats and the GREENS go from 0 to 14 seats.
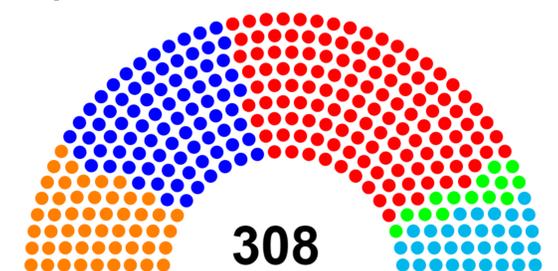

Figure 9: Simulated 2008: CONSERVATIVES 82 seats; LIBERALS 129; BLOC 32; NDP 51; GREENS 14

Strategic PR takeaway: Modeling strategic voting can dramatically change the expected result of electoral system reform.

Conclusions

To realistically capture the results of electoral system change, we cannot neglect strategic voting. One Canadian party, the LIBERALS, had a chance to legislate on the issue. They support RV, but it only conditionally helps them. They oppose PR, but voter strategy suggests PR might benefit them. My method can be applied to other SMD cases, like the UK and the US.

Works cited

[1] Walter Mebane, Samuel Baltz, and Fabricio Vasselai, "Using Agent-Based Models to Simulate Strategic Behavior in Elections" (2019)
[2] Gary Cox, "Strategic Voting Equilibria Under the Single Nontransferable Vote" (1994)